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Numerical solutions of Fingero-Imbibition in a homogeneous medium involving magnetic fluid

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Abstract:

The present paper numerically discusses the phenomenon of fingero-imbibition in a double phase displacement process through homogeneous porous medium with involvement of a layer of magnetic fluid in the injected phase. This phenomenon have much importance particularly in petroleum technology and hydrology. The basic equations of the flow system coupled with analytical consideration for additional physical effects yields a nonlinear partial differential equation whose numerical solution has been obtained by reduced differential transform method. The graphs of the numerical results indicate the stabilization of fingers.

Key words: fingero-Imbibition phenomenon, Homogeneous porous medium, RDTM

Introduction

The present paper deals with the phenomena of Fingero-Imbibition in double phase flow through homogeneous porous media involving magnetic fluid. This phenomenon arises on account of simultaneous occurrence of two important phenomena imbibition and fingering. We have assumed that injection of preferentially wetting, less viscous fluid into porous medium saturated with resident fluid is initiated under imbibition and in consequence the resident fluid is pushed by drive in secondary recovery process. Verma called these conjoint phenomena as Fingero-Imbibition. It is well known that when a porous medium filled with some resident fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to the difference in wetting abilities is called counter-current imbibition. Similarly, when a fluid contained in a porous medium is displaced by another fluid of lesser viscosity, instead of regular displacement of whole front, protuberances (fingers) may occur which shoot through the porous medium at relatively great speeds. This phenomenon is called fingering or instabilities. The phenomena of fingering and imbibition occurring simultaneously in displacement process, have gained much current importance due to their frequent occurrence in the problem of petroleum technology and many authors have discussed them from different point of view. In this paper, the underlying assumptions are that the two fluids are immiscible and the injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure. The mathematical formulation of basic equations yields a Non-linear partial differential equation governing with fingero-imbibition in the investigated liquid-liquid displacement problem. A numerical solution is obtained by Successive over Relaxation Method.

Statement of the Problem:

We consider here a finite cylindrical mass of porous medium of length L(=1) saturated with native liquid (o), is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibition face (x=0) and this end is exposed to an adjacent formation of 'injected' liquid (w) which involves a thin layer of suitable magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (w) is initiated by imbibition and the consequent displacement of native liquid (o) produces protuberances (fingers). This arrangement describes a one – dimensional phenomenon of Fingero-Imbibition.

Formulation of the Problem:

Assuming that the flow of two immiscible phases is governed by Darcy's law, we may write the seepage velocity of injected and native fluid as,

1) $V_w = -\left(\frac{K_w}{\partial_w}\right) K \left[\frac{\partial p_w}{\partial w} + \gamma H \frac{\partial H}{\partial x}\right]$

2) $V_o = -\left(\frac{K_o}{\partial_o}\right) K \left[\frac{\partial p_o}{\partial x}\right]$

Where

$$\gamma = \mu_o \lambda + \frac{16\lambda \mu_0 \lambda^2 r^3}{g(t+2)^3}$$

K = The permeability of the homogeneous medium

K_w = Relative permeability of injected fluid, which is function of S_w

K_o = Relative permeability of injected fluid, which is function of S_o

S_w = The saturation of injected fluid

S_o = The saturation of native fluid

P_w = Pressure of injected fluid

P_o = Pressure of native fluid

δ_w, δ_o = Constants kinematics viscosities

g = Acceleration due to gravity.

Neglecting the variation in phase densities, the equation of continuity for injected fluid can be written as:

3) $p\left(\frac{\partial S_w}{\partial t}\right) + \left(\frac{\partial V_w}{\partial x}\right) = 0$

Where p is porosity of the medium.

From the definition of phase saturation it is obvious that $S_w + S_o = 1$. The analytical condition (Scheidegger, 1960) governing imbibition phenomenon is

4) $V_w + V_o = 0$

From the definition of capillary pressure P_c as the pressure discontinuity between two phases yields

5) $P_c = P_o - P_w$

Substituting the values of V_o and V_w from equations (1) & (2) into equation (4), we get,

6) $\left(\frac{K_w}{\delta_w}\right) K\left[\frac{\partial P_w}{\partial x} + \gamma H \frac{\partial H}{\partial x}\right] + \left(\frac{K_o}{\delta_o}\right) K\left[\frac{\partial P_o}{\partial x}\right] = 0$

Thus equation (6) reduces to the form,

7) $\frac{\partial P_w}{\partial x} = \frac{[\left(\frac{K_o}{\delta_o}\right)\frac{\partial P_c}{\partial x} + \left(\frac{K_w}{\delta_w}\right)\gamma H \frac{\partial H}{\partial x}]}{\left(\frac{K_o}{\delta_o} + \frac{K_w}{\delta_w}\right)}$

Equation (1) together with (7) yields

8) $V_w = \left(\frac{K_w K_o}{K_o \delta_w + K_w \delta_o}\right) K\left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right] = 0$

Substituting equation (8) into (3) we get,

9) $p\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial}{\partial x}\left[\left(\frac{K_w K_o}{K_o \delta_w + K_w \delta_o}\right) K\left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right]\right] = 0$

This is the desired non-linear differential equation describing the finger-imbibition phenomenon for the flow of two immiscible phases through porous media.

Since the present investigation involves injected fluid and a viscous native fluid, therefore according to Scheidegger (1960) approximation, we may write equation (9) in the form

10) $p\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial}{\partial x}\left[\left(\frac{K_o}{\delta_o}\right) K\left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right]\right] = 0$

As $\left(\frac{K_w K_o}{K_o \delta_w + K_w \delta_o}\right) \approx \frac{K_o}{\delta_o}$

As this stat, for definiteness of the mathematical analysis, we assume standard relationship due to Scheidegger [1], Muskat [11], between phase saturation and relative permeability as

11) $K_w = S_w, S_o = 1 - S_w$ and $P_c = -\beta_o S_w$

Where β_o is capillary pressure coefficient. Substituting the values from equation (11) into (10) we get,

12) $p\left(\frac{\partial S_w}{\partial t}\right) + \frac{K}{\delta_o}\frac{\partial}{\partial x}\left((1 - S_w)\left[-\beta_o \frac{\partial S_w}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right]\right) = 0$

Considering the magnetic fluid H in the x-direction only, we may write [27], $H = \frac{\Lambda}{x^n}$ where Λ is a constant parameter and n is an integer. Using the value of H for $n = -1$ in equation (12), we get,

$$p\left(\frac{\partial S_w}{\partial t}\right) + \frac{K}{\delta_o}\frac{\partial}{\partial x}\left((1 - S_w)\left[-\beta_o \frac{\partial S_w}{\partial x} - \gamma\lambda^2 x\right]\right) = 0$$

or

13) $p\left(\frac{\partial S_w}{\partial t}\right) - \frac{K\beta_o}{\delta_o}\frac{\partial}{\partial x}\left((1 - S_w)\frac{\partial S_w}{\partial x}\right) - \frac{K\gamma\lambda^2}{\delta_o}\frac{\partial}{\partial x}\left((1 - S_w)x\right) = 0$

A set of suitable initial and boundary conditions associated to problem (13) are

14) $S_w(x, 0) = S_c$ for all $x > 0$

15) $S_w(0,t) = S_{wi}$ $S_w(L,t) = S_1$ for all $t \geq 0$

Equation (13) is reduced to dimensionless form by setting

$$X = \frac{x}{L}, \quad T = \frac{K\beta t}{\delta_0 L^2 p}, S^*_w(x,t) = 1 - S_w(x,t)$$

And then equation (13) takes the form

16) $\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left(S_w \frac{\partial S_w}{\partial x}\right) - C_0 \frac{\partial}{\partial x}(S_w x)$

Where $C_0 = \frac{\gamma \wedge^2 L^2}{\beta_0}$

With auxiliary conditions

$S_w(x,0) = 1 - S_c \quad 0 < x \leq L \quad (16(a))$

$S_w(0,t) = 1 - S_o \quad$ for all t $\quad (16(b))$

$S_w(L,t) = 1 - S_1 \quad$ for all t $\quad (16(c))$

In equation (16), the asterisks are dropped for simplicity. Equation (16) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium with effect of magnetic fluid.

The problem is solved by using Differential Transform method. The numerical values are shown by table. Curves indicate the behavior of saturation of water corresponding to various time periods.

Solution Using RDTM Method

$$\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left(S_w \frac{\partial S_w}{\partial x}\right) - C_0 \frac{\partial}{\partial x}(S_w x)$$

Taking the initial condition $S_w(x,0) = S_{w0} = f(x)$

17) $f(x) = \frac{e^x - 1}{e - 1}$

The problem is solved by reduced differential transform method because our equation is non-linear partial differential equation.

Reduced differential Transform Method

The Basic definition of RDTM is given below

If the function u(x,t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let

$$U_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]$$

Where the t-dimensional spectrum function $U_k(x)$ is the transformed function . u(x,t) represent transformed function. The differential inverse transform of $U_k(x)$ is defined as follow

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k$$

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right] t^k$$

Apply RDTM on (16)

18) $(k+1)S_{w(k+1)}(x) = \left[\sum_{r=0}^{k} (S_{wr})(S_{w(k-r)})_{xx}\right] + [(S_{wk})_x)^2] \quad - C_0([(S_{wk})_x(kx)] + S_{wk}(x))$

Now let $k = 0$ & $C_0 = 1$ then put initial condition (17) into eq. (16). So we get $S_{w1}(x)$ as following

$$(1)S_{w1}(x) = [(S_{w0})(S_{w0})_{xx}] + [((S_{w0})_x)^2] - [[(S_{w0})_x(0)x] + S_{w0}(x)]$$

$$S_{w1}(x) = \left(\left(\frac{e^x-1}{e-1}\right)\left(\frac{e^x}{e-1}\right)\right) + \left(\left(\frac{e^x}{e-1}\right)^2\right) - \left(0 + \left(\frac{e^x-1}{e-1}\right)\right)$$

$$S_{w1}(x) = \frac{2e^{2x} - e^{x+1} + e - 1}{(e-1)^2}$$

For second iteration let $k = 1$

$$(k+1)S_{w(k+1)}(x) = \left[\sum_{r=0}^{1}(S_{wr})(S_{w(k-r)})_{xx}\right] + [((S_{wk})_x)^2] - ([(S_{wk})_x(kx)] + S_{wk}(x))$$

$$(2)S_{w2}(x) = \left[(S_{w0})(S_{w(1-0)})_{xx} + (S_{w1})(S_{w(1-1)})_{xx}\right] + [((S_{w1})_x)^2] - ([(S_{w1})_x(1.x)] + S_{w1}(x))$$

$$(2)S_{w2}(x) = \left(\left(\frac{e^x-1}{e-1}\right)\left(\frac{8e^{2x}-e^{x+1}}{(e-1)^2}\right) + \left(\frac{2e^{2x}-e^{x+1}+e-1}{(e-1)^2}\right)\left(\frac{e^x}{e-1}\right)\right) + \left(\frac{4e^{2x}-e^{x+1}}{(e-1)^2}\right)^2 - \left[\left(\frac{4e^{2x}-e^{x+1}}{(e-1)^2}\right)x + \left(\frac{2e^{2x}-e^{x+1}+e-1}{(e-1)^2}\right)\right]$$

$$= \frac{2e3^{x+1}+(8x-2)e^{2x+1}-(4x+3)e^{2(x+1)}-10e^{3x}+(6-4x)e^{2x}+(x-2)e^{x+1}+e^{x}+16e^{4x}+(x+1)e^{x+3}-e^3+3e^2-3e+1-2xe^{x+2}}{(e-1)^4}$$

In this way we can generated other polynomials by putting different values in equation (18)

Now by inverse Transform

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$$

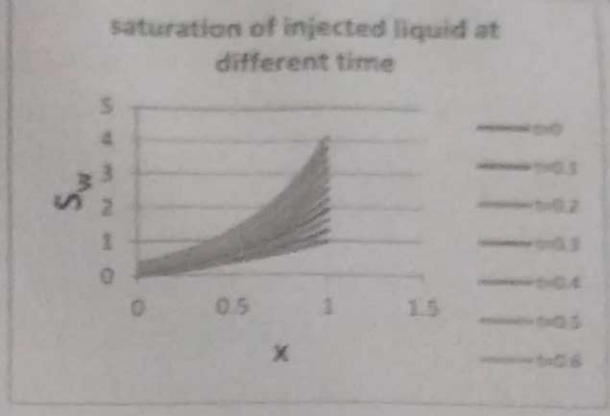
$$S_w(x,T) = S_{w0}(x)T^0 + S_{w1}(x)T^1 + S_{w2}(x)T^2 + \dots$$

$$S_w(x,t) = \frac{e^x-1}{e-1}T^0 + \frac{2e^{2x}-e^{x+1}+e-1}{(e-1)^2}T^1 + \frac{2e3^{x+1}+(8x-2)e^{2x+1}-(4x+3)e^{2(x+1)}+(x-2)e^{x+1}+(x+1)e^{x+3}-2xe^{x+2}}{(e-1)^4}\frac{T^2}{2} + \frac{16e^{4x}-10e^{3x}+(6-4x)e^{2x}+e^x-e^3+3e^2-3e+1}{(e-1)^4}\frac{T^2}{2}\dots$$

Table And Figure

The following table shows the approximate solution for saturation of injected liquid for different values of x at different time using RDTM

x	t=0	t=0.1	t=0.2	t=0.3	t=0.4	t=0.5	t=0.6	t=0.7	t=0.8	t=0.9	t=1
0	0	0.03387	0.067739	0.101609	0.135479	0.169348	0.203218	0.237088	0.270958	0.304827	0.338697
0.1	0.061307	0.100192	0.139576	0.178761	0.217945	0.25713	0.296314	0.335499	0.374684	0.413868	0.453053
0.2	0.128851	0.175653	0.222454	0.269256	0.316058	0.362859	0.409661	0.456463	0.503264	0.550066	0.596867
0.3	0.20361	0.260959	0.318308	0.375656	0.433005	0.490354	0.547703	0.605052	0.662401	0.71975	0.777099
0.4	0.286231	0.357837	0.429443	0.501049	0.572655	0.644261	0.715867	0.787473	0.859079	0.930685	1.002291
0.5	0.377541	0.46808	0.558619	0.649158	0.739697	0.830236	0.920775	1.011313	1.101852	1.192391	1.28293
0.6	0.478454	0.593797	0.709139	0.824482	0.939825	1.055167	1.17051	1.285853	1.401195	1.516538	1.631881
0.7	0.58998	0.737474	0.884967	1.032461	1.179954	1.327448	1.474941	1.622435	1.769928	1.917422	2.064915
0.8	0.713238	0.90205	1.090863	1.279676	1.46849	1.657303	1.846116	2.03493	2.223743	2.412556	2.60137
0.9	0.849455	1.091003	1.332551	1.574099	1.815646	2.057194	2.298742	2.54029	2.781838	3.023386	3.264934
1	1	1.308463	1.616925	1.925388	2.233851	2.542314	2.850776	3.159239	3.467702	3.776164	4.084627



Conclusion

In graph, X-axis represents the different values of x and Y-axis represents saturation of injected liquid in saturated porous media. It is clear from graph that, for each value of t, saturation of injected liquid involving magnetic fluid increases with increases in value of x and saturation is also increases with each value of x when time is also increases.

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Numerical Solutions of Fingero-Imbibition in a Slightly Dipping Porous Media Involving Magnetic Fluid

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Abstract: The present paper numerically discusses the phenomenon of fingero-imbibition in a double phase displacement process through slightly dipping porous medium with involvement of a layer of magnetic fluid in the injected phase. This phenomenon is assumed to occur in a double-phase displacement process involving two immiscible liquids of small viscosity difference in which the injection is initiated by imbibitions and the consequential displacement of the relatively more viscous native liquid produces fingers. The basic equations of the flow system coupled with analytical consideration for additional physical effects yields a nonlinear partial differential equation whose numerical solution has been obtained by reduced differential transform method.

Keywords: fingero-Imbibition phenomenon, Homogeneous porous medium, RDTM

I. Introduction

The present paper deals with the phenomena of Fingero-Imbibition in double phase flow through porous homogeneous, cracked and slightly dipping porous media involving magnetic fluid. This phenomenon arises on account of simultaneous occurrence of two important phenomena imbibition and fingering. We have assumed that injection of preferentially wetting, less viscous fluid into porous medium saturated with resident fluid is initiated under imbibition and in consequence, the resident fluid is pushed by drive in secondary recovery process. Verma called these conjoint phenomena as Fingero-Imbibition. The phenomena of fingering and imbibition occurring simultaneously in displacement process, have gained much current importance due to their frequent occurrence in the problem of petroleum technology and many authors have discussed them from different point of view. In this paper, the underlying assumptions are that the two fluids are immiscible and the injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure. The mathematical formulation of basic equations yields a Non-linear partial differential equation governing with fingero-imbibition in the investigated liquid-liquid displacement problem. A numerical solution is obtained by Successive over Relaxation Method.

Statement of the Problem:

We consider here a finite cylindrical mass of porous medium of length $L(=1)$ saturated with native liquid (o), is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibition face $(x=0)$ and this end is exposed to an adjacent formation of 'injected' liquid (w) which involves a thin layer of suitable magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (w) is initiated by imbibition and the consequent displacement of native liquid (o) produces protuberances (fingers). This arrangement describes a one – dimensional phenomenon of Fingero-Imbibition. The cylindrical of porous matrix is inclined at small angle θ with horizontal only and remaining process of Fingero-Imbibition goes as it is.

Formulation of the Problem:

Assuming that the flow of two immiscible phases is governed by Darcy's law, we may write the seepage velocity of injected and native fluid as,

$$V_w = -\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial w} + \gamma H\frac{\partial H}{\partial x} + \rho_w g\sin\theta\right] \tag{1}$$

$$V_o = -\left(\frac{K_o}{\delta_o}\right)K\left[\frac{\partial P_o}{\partial x} + \rho_o g\sin\theta\right] \tag{2}$$

Where

$$\gamma = \mu_o \lambda + \frac{16\pi\mu_o \lambda^2 r^3}{g(t+2)^3}$$

K = The permeability of the homogeneous medium
K_w = Relative permeability of injected fluid, which is function of S_w
K_o = Relative permeability of injected fluid, which is function of S_o
S_w = The saturation of injected fluid
S_o = The saturation of native fluid
P_w = Pressure of injected fluid
P_o = Pressure of native fluid
g = Acceleration due to gravity.

Where the phase densities ρ_w & ρ_o are regarded as constant, δ_w and δ_o are assumed to be invariant to magnetic field, and θ is inclination.

Neglecting the variation in phase densities, the equation of continuity for injected fluid can be written as:

$$p\left(\frac{\partial S_w}{\partial t}\right) + \left(\frac{\partial V_w}{\partial x}\right) = 0 \qquad (3)$$

Where p is porosity of the medium.

From the definition of phase saturation it is obvious that $S_w + S_o = 1$. The analytical condition (Scheidegger, 1960) governing imbibition phenomenon is

$$V_w + V_o = 0 \qquad (4)$$

From the definition of capillary pressure P_c as the pressure discontinuity between two phases yields

$$P_c = P_o - P_w \qquad (5)$$

On Simplifying Equation (1)&(2) by using Equation (4) & (5) we obtain

$$\frac{\partial P_o}{\partial t} = \left[\frac{-K_w\delta_o\left[\gamma H\frac{\partial H}{\partial x} + \rho_w g\sin\theta\right] - K_o\delta_w\left[\frac{\partial P_o}{\partial x} + \rho_o g\sin\theta\right]}{\left(\frac{K_o}{\delta_o} + \frac{K_w}{\delta_w}\right)}\right]\left(\frac{\partial P_c}{\partial x}\right) \qquad (6)$$

Substituting above equation into equation (1), we obtain

$$V_o = \frac{KK_oK_w}{(K_o/\delta_o + K_w/\delta_w)}\left[\frac{\partial P_c}{\partial x} - \gamma H\frac{\partial H}{\partial x} + (\rho_o - \rho_w)g\sin\theta\right] \qquad (7)$$

Since $\frac{K_oK_w}{(K_o/\delta_o + K_w/\delta_w)} \approx \frac{K_o}{\delta_o}$, Above equation reduces to the form

$$V_o = \frac{KK_o}{\delta_o}\left[\frac{\partial P_c}{\partial x} - \gamma H\frac{\partial H}{\partial x} + (\rho_o - \rho_w)g\sin\theta\right] \qquad (8)$$

Substituting equation (8) into (3) we get,

$$p\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial}{\partial x}\left(\frac{KK_o}{\delta_o}\left[\frac{\partial P_c}{\partial x} - \gamma H\frac{\partial H}{\partial x} + (\rho_o - \rho_w)g\sin\theta\right]\right) = 0 \qquad (9)$$

At this state, for definiteness of the mathematical analysis, we assume standard relationship due to Scheidegger and Johnson [1], Muskat [2], between phase saturation and relative permeability as

$$K_w = S_w, S_o = 1 - S_w \text{ and } P_c = -\beta_o S_w$$

Where β_o is capillary pressure coefficient. Considering the magnetic fluid H in the x-direction only, we may write [3], $H = \frac{\Lambda}{x^n}$ where Λ is a constant parameter and n is an integer. Using the value of H for $n = -1$. Substituting all values in equation (9), we get,

$$p\left(\frac{\partial S_w}{\partial t}\right) + \frac{K}{\delta_o}\frac{\partial}{\partial x}\left((1 - S_w)\left[-\beta_o\frac{\partial S_w}{\partial x} - \gamma\Lambda^2 x + (\rho_o - \rho_w)g\sin\theta\right]\right) = 0 \qquad (10)$$

A set of suitable initial and boundary conditions associated to equation (10) are

$$S_w(x,0) = S_1 \text{ for all } x > 0 \qquad (11)$$

$$S_w(0,t) = S_{w_0};\quad S_w(L,t) = S_{w_1} \quad \text{for all } t \geq 0 \qquad (12)$$

Equation (10) is reduced to dimensionless form by setting

$$X = \frac{x}{L},\quad T = \frac{t}{L^2(C_1/C_2)}, S_w(x,t) = 1 - S_w(x,t)$$

So that

$$\frac{\partial S_w}{\partial T} - \frac{\partial}{\partial x}\left(S_w\frac{\partial S_w}{\partial x}\right) + C_0\frac{\partial}{\partial x}(S_w x) - C_1\left(\frac{\partial S_w}{\partial x}\right) = 0 \qquad (13)$$

Where $C_0 = \frac{\alpha\Lambda^2 L^2}{\beta_0}$, $C_1 = \frac{L}{\beta_0}(\rho_o - \rho_w)g\sin\theta$. Asterisks are dropped for simplicity.

With auxiliary

$$S_w(x,0) = 1 - S_1 \qquad \text{for all } x > 0 \qquad (13(a))$$

$$S_w(0,t) = 1 - S_{w_0} \qquad for\ all\ t \geq 0 \qquad (13(b))$$

$$S_w(L,t) = 1 - S_{w_1} \qquad for\ all\ t \geq 0 \qquad (13(c))$$

Equation (13) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium with effect of magnetic fluid.

The problem is solved by using Differential Transform method. The numerical values are shown by table. Curves indicate the behavior of saturation of water corresponding to various time periods.

II. Solution Using Rdtm Method

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial x}\left(S_w \frac{\partial S_w}{\partial x}\right) - C_0 \frac{\partial}{\partial x}(S_w x) + C_1\left(\frac{\partial S_w}{\partial x}\right) \qquad (13)$$

$$\therefore (S_w)_T = (S_w (S_w)_x)_x - C_0 (S_w x)_x + C_1 (S_w)_x$$

Taking the initial condition $S_w(x,0) = S_{w0} = f(x)$

$$f(x) = \frac{e^x - 1}{e - 1} \qquad (14)$$

The problem is solved by reduced differential transform method because our equation is non-linear partial differential equation.

Reduced differential Transform Method

The Basic definition of RDTM is given below

If the function u(x,t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let

$$U_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]$$

Where the t-dimensional spectrum function $U_k(x)$ is the transformed function . u(x,t) represent transformed function. The differential inverse transform of $U_k(x)$ is defined as follow

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k$$

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right] t^k$$

Apply RDTM on (13)

$$(k+1) S_{w(k+1)}(x) = \left[\sum_{r=0}^{k} (S_{wr})(S_{w(k-r)})_{xx}\right] + \ [(S_{wk})_x)^2] - C_0([(S_{wk})_x(kx)] + S_{wk}(x)) + C_1 (S_{wk})_x \qquad (15)$$

Now let $k = 0$ & $C_0 = 1, C_1 = 1$ then put initial condition (14) into eq. (15), So we get $S_{w1}(x)$ as following

$$(1) S_{w1}(x) = [(S_{w0})(S_{w0})_{xx}] + [((S_{w0})_x)^2] - [[(S_{w0})_x(0)x] + S_{w0}(x)] + ((S_{w0})_x$$

$$S_{w1}(x) = \left(\left(\frac{e^x - 1}{e - 1}\right)\left(\frac{e^x}{e - 1}\right)\right) + \left(\left(\frac{e^x}{e - 1}\right)^2\right) - \left(0 + \left(\frac{e^x - 1}{e - 1}\right) + \left(\frac{e^x}{e - 1}\right)\right)$$

$$S_{w1}(x) = \frac{2e^{2x} - e^x + e - 1}{(e - 1)^2}$$

For second iteration let $k = 1$

$$(k+1) S_{w(k+1)}(x) = \left[\sum_{r=0}^{1} (S_{wr})(S_{w(k-r)})_{xx}\right] + [(S_{wk})_x)^2] - ([(S_{wk})_x(kx)] + S_{wk}(x)) + (S_{wk})_x$$

$$(2) S_{w2}(x) = \left[(S_{w0})(S_{w(1-0)})_{xx} + (S_{w1})(S_{w(1-1)})_{xx}\right] + [(S_{w1})_x)^2] - ([(S_{w1})_x(1.x)] + S_{w1}(x)) + (S_{w1})_x$$

$$(2) S_{w2}(x) = \left(\left(\frac{e^x - 1}{e - 1}\right)\left(\frac{8e^{2x} - e^x}{(e - 1)^2}\right) + \left(\frac{2e^{2x} - e^x + e - 1}{(e - 1)^2}\right)\left(\frac{e^x}{e - 1}\right)\right) + \left(\frac{4e^{2x} - e^x}{(e - 1)^2}\right)^2 - \left[\left(\frac{4e^{2x} - e^x}{(e - 1)^2}\right)x + \left(\frac{2e^{2x} - e^x + e - 1}{(e - 1)^2}\right)\right] + \left(\frac{4e^{2x} - e^x}{(e - 1)^2}\right)$$

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$$= \frac{10e^{3x+1}-(14-8x)e^{2x+1}-(2-4x)e^{2(x+1)}-18e^{3x}+(13-4x)e^{2x}-(1+2x)e^{x+1}+xe^{x}+16e^{4x}+(x+1)e^{x+2}-e^{3}+3e^{2}-3e+1}{(e-1)^4}$$

In this way we can generated other polynomials by putting different values in equation (18)
Now by inverse transform

$$u(x,t) = \sum_{k=0}^{r} U_k(x)t^k$$

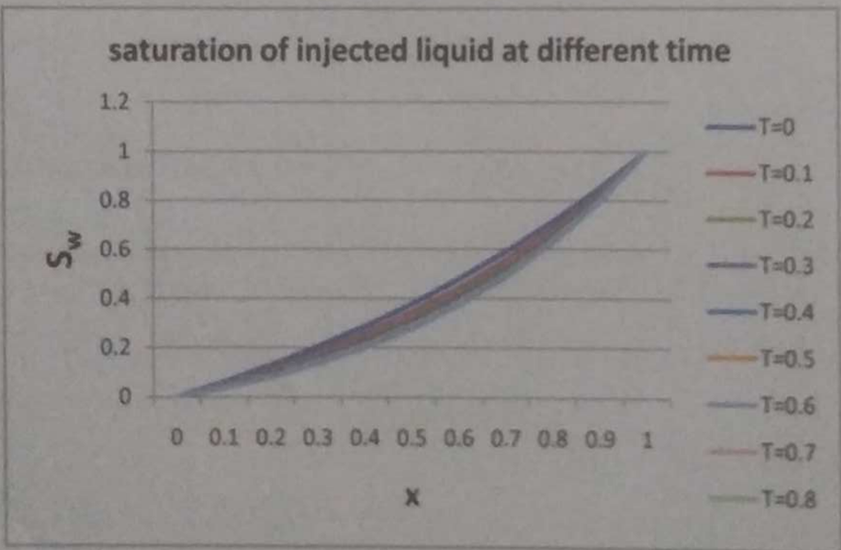
$$S_w(x,T) = S_{w0}(x)T^0 + S_{w1}(x)T^1 + S_{w2}(x)T^2 + \cdots$$

$$S_w(x,t) = \frac{e^x-1}{e-1}T^0 + \frac{2e^{2x}-e^x+e-1}{(e-1)^2}T^1 + \frac{10e^{3x+1}-(14-8x)e^{2x+1}-(2-4x)e^{2(x+1)}-(1+2x)e^{x+1}+(x+1)e^{x+2}}{(e-1)^4}\frac{T^2}{2} + \frac{16e^{4x}-18e^{3x}+(13-4x)e^{2x}+xe^{x}-e^3+3e^2-3e+1}{(e-1)^4}\frac{T^2}{2}\cdots$$

III. Table And Figure

The following table shows the approximate solution for saturation of injected liquid for different values of x at different time using RDTM

X	T=0	T=0.1	T=0.2	T=0.3	T=0.4	T=0.5	T=0.6	T=0.7	T=0.8	T=0.9	T=1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.0612	0.0528	0.048	0.0449	0.0428	0.0412	0.04	0.039	0.0382	0.0375	0.037
0.2	0.1289	0.1125	0.1032	0.0971	0.0929	0.0898	0.0874	0.0855	0.0839	0.0826	0.0815
0.3	0.2036	0.18	0.1665	0.1578	0.1517	0.1472	0.1437	0.141	0.1387	0.1368	0.1353
0.4	0.2862	0.2565	0.2395	0.2285	0.2208	0.2151	0.2107	0.2073	0.2044	0.2021	0.2001
0.5	0.3775	0.3433	0.3238	0.3111	0.3023	0.2958	0.2907	0.2867	0.2834	0.2807	0.2785
0.6	0.4785	0.4421	0.4214	0.4079	0.3986	0.3916	0.3862	0.382	0.3785	0.3757	0.3733
0.7	0.59	0.5548	0.5346	0.5216	0.5125	0.5058	0.5006	0.4965	0.4931	0.4903	0.488
0.8	0.7132	0.6835	0.6664	0.6554	0.6477	0.6421	0.6377	0.6342	0.6313	0.629	0.627
0.9	0.8495	0.8308	0.8202	0.8133	0.8085	0.805	0.8022	0.8001	0.7982	0.7968	0.7956
1	1	1	1	1	1	1	1	1	1	1	1



IV. Conclusion

In the graph X-axis represents the values of x and Y-axis represents the saturation of injected liquid involving magnetic fluid sw in porous media of length one. It is clear from graph that, at particular time, saturation of injected liquid involving magnetic fluid decrease with increase in value of x (or as we move ahead) and at x=1, saturation is decreased to zero and as time increases, rate of increase of the saturation of injected liquid decreases at each layer.

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Numerical Solutions of Burger's Equation Arising in Longitudinal Dispersion Phenomenon in Fluid Flow Through Porous Media by Reduced Differential Transform Method

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ABSTRACT: The longitudinal dispersion phenomenon of miscible phenomenon of miscible fluids in porous media is discussed in which the flow is unsteady. After the mathematical formulation of the phenomenon get a non-linear partial differential equation explicitly Burger's equation. We get the series solution of the problem by using Reduced differential transform method (RDTM) & also derived numerical solution by MATLAB. This type of phenomenon has been of great importance to hydrologist & in oil industries.

KEYWORDS: Miscible fluids, longitudinal dispersion phenomenon, Burger's equation, RDTM.

I. INTRODUCTION

The present paper discuses the solution of longitudinal dispersion phenomenon, which arising in the miscible fluid flow through homogeneous porous media. In a miscible displacement process a fluid is displaced in a porous medium by another fluid that is miscible with the first fluid. Miscible displacement in porous media plays an important role in many engineering and science fields. Among many flow problems in porous media, one involves fluid mixtures called miscible fluids. A miscible fluid is a single phase fluid consisting of several completely dissolved homogeneous fluid species, a distinct fluid-fluid interface doesn't exist in a miscible fluid. So that the interfacial tension between them is zero. The problems of dispersion have been receiving considerable attention from chemical, environmental and petroleum engineers, hydrologist, mathematicians and soil scientists. Over the past ten decades longitudinal dispersion in porous media has been studied and correlated extensively for gaseous and aqueous systems.

II. RELATED WORK

Several researchers have discussed this problem with common assumption of homogeneous porous media with constant porosity, steady seepage flow velocity and constant dispersion coefficient but their techniques are different such as P.H.Bhathawala [4] derived analytical solution of miscible fluid flow through homogeneous porous media by applying a two parameter singular perturbation method. K.J.Chauhan, Falguni Dabhade & D.M. Patel [5] discussed analytical solution of the longitudinal dispersion problem by using infinitesimal transformations group technique of similarity analysis. R.Meher, S.K.Meher & M.N.Mehta [6, 7] studied numerical & graphical solution of dispersion phenomenon by Adomain decomposition method and they also employed a New approach to Backlund transformation for longitudinal dispersion of miscible fluid flow through porous media in oil reservoir during secondary recovery process. Ravi N. Borana, V.H.Pradhan & M.N.Mehta [8, 9] obtained Numerical solution of Burger's equation by using finite difference method and they also discussed numerical solution of Burger's equation in longitudinal dispersion phenomenon in fluid flow through porous media by Cranck-nicolson scheme. Kunjan shah & Twinkle Singh [10, 11]

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solved Burger's equation in one dimensional ground water recharge by spreading numerically by q-Homotopy Analysis method & they also derived Mixture of New Integral transform & Homotopy perturbation method to solve Burger's equation arising in the longitudinal dispersion phenomenon. R. Meher & M.N.Mehta [12] discussed a new approach to backlund transformations to solve Burger's equation. Kajal patel, M.N.Mehta & Twinkle R. singh [13] studied a solution of one-dimensional dispersion phenomenon by Homotopy Analysis Method. Rudraiah et al. [14] have provided analytical study of the dispersion in saturated deformable or non-deformable porous media with or without chemical reaction, considering a series of particular cases selected through different practical problems using different dispersion models. They obtained basic equation using mixture and homogenation. Different analytical and numerical models are valid for long time (asymptotic) and for all time (transient) are explained.

III. STATEMENT OF THE PROBLEM

Miscible displacement in porous media is a type of double-phase flow in which the two phases are completely soluble in each other. Therefore capillary forces between the fluids do not come into effect. The longitudinal dispersion of the impure or saline water with the concentration c(x, t) flowing in the x-direction has been considered, that the ground water recharge takes place over a large basin contain homogeneous porous medium is saturated with fresh water. The miscible flow under conditions of complete miscibility could be thought to behave locally at least, as a single-phase fluid, which would obey Darcy's law. The change of concentration, in turn, would be caused by the bulk coefficients of diffusion of the one fluid in the other. There is no mass transfer between solid and liquid phases, is assumed [2]. The miscible flow takes place both longitudinally and transversely, but the spreading caused by dispersion is greater in the direction of flow than the transverse direction. One dimensional treatment of these systems avoids treatment of a radial or transverse component of dispersion. We only consider the dispersion phenomenon in the direction of flow i.e. longitudinal dispersion, which takes places when miscible fluids flow in homogeneous porous media. The problem is to describe the growth of mixed region. i.e. to find concentration c(x, t) of the impure water as a function of time t and position x, as two miscible fluids flow through homogeneous porous media. Outside of the mixed zone (on either side) the single fluid equation describes the motion of fluid.

IV. FORMULATION OF THE PROBLEM

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \bar{v}) = 0 \qquad (1)$$

Where 'ρ' is the density for mixture and '$\bar{v}$' is the pore seepage velocity vector.
The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial C}{\partial t} + \nabla.(C\bar{v}) = \nabla.\left[\rho \bar{D} \nabla\left(\frac{C}{\rho}\right)\right] \qquad (2)$$

Where 'C' is the concentration of a fluid in a porous media. D is the coefficient of dispersion with nine components D_{ij}. In a laminar flow for an Incompressible fluid through homogeneous porous medium, density 'ρ' is constant. Then equation (2) becomes,

$$\frac{\partial C}{\partial t} + \bar{v}.(\nabla C) = \nabla.[\bar{D}\nabla C] \qquad (3)$$

Let us assume that the seepage velocity $\bar{v}$ is along the x-axis, then $\bar{v} = u(x,t)$ and the non zero components will be $D_{11} \approx D_L = \gamma$ (coefficient of longitudinal dispersion) and other Components will be zero [6].
Equation (3) becomes,

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \gamma \frac{\partial^2 C}{\partial x^2} \qquad (4)$$

Where u is the component velocity along x-axis which is time dependent as well as concentration along x-axis in $x \geq 0$ direction and $D_L > 0$ and it is the cross sectional flow velocity in porous media.
$u = \frac{C(x,t)}{C_0}$, Where $x > 0$ *and for* $C_0 \cong 1$ by [6]. Equation (4) becomes

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$$\frac{\partial C}{\partial t}+C\frac{\partial C}{\partial x}-\gamma\frac{\partial^2 C}{\partial x^2}=0 \qquad (5)$$

Where γ is the coefficient of the longitudinal dispersion.
This is the non linear Burger's equation for longitudinal dispersion of miscible fluid flow through porous media. The theory that follows is confined to dispersion in unidirectional seepage flow through semi finite homogeneous porous media. The seepage flow velocity of impure water is assumed unsteady. Here the initial concentration of dispersion is considered as an input highest constant concentration of impurity at $x = 0$ is C_0. The porous medium is considered as nonadsorbing.
The governing partial differential equation (5) for longitudinal hydrodynamic dispersion with in a semi finite nonadsorbing porous medium in a unidirectional flow field in which γ is the longitudinal dispersion coefficient, C is the average cross-sectional concentration, u is the unsteady seepage velocity, x is a coordinate parallel to flow and t is time.
We can write equation (7) as $C(x,t) = \gamma C_{xx} - C\,C_x$
The problem is solved by using Reduced Differential Transform method. The numerical values are shown by table. Curves indicate the moisture content corresponding to various time periods.

V. PROPOSED ALGORITHM

$$\frac{\partial C}{\partial t}+C\frac{\partial C}{\partial x}-\gamma\frac{\partial^2 C}{\partial x^2}=0 \qquad (5)$$

Taking $\gamma = 1$ & the initial condition $C(x,0) = C_0 = f(x)$

$$f(x) = e^{-x} \qquad (6)$$

The problem is solved by reduced differential transform method because our equation is partial differential equation.

Reduced differential Transform Method
The Basic definition of RDTM is given below
If the function u(x, t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let

$$U_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]$$

Where the t-dimensional spectrum function $U_k(x)$ is the transformed function. u(x,t) represent transformed function. The differential inverse transform of $U_k(x)$ is defined as follow

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$$

$$u(x,t) = \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]t^k$$

Apply RDTM on (5)

$$(k+1)C_{(k+1)}(x,t) = ((C_k)_{xx} - \left[\sum_{r=0}^{k}(C_r)(C_{(k-r)})_x\right] \qquad (7)$$

Now let k=0 then put initial condition (6) into eq. (7). So we have the values of $C_k(x)$ as following

$$(1)C_1(x,t) = ((C_0)_{xx} - \left[\sum_{r=0}^{0}(C_r)(C_{(k-r)})_x\right]$$

$$C_1(x,t) = ((C_0)_{xx} - \left[(C_0)(C_{(0-0)})_x\right]$$

$$C_1(x,t) = e^{-x} - [(e^{-x})(-e^{-x})]$$

$$C_1(x,t) = e^{-x} + e^{-2x}$$

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Now let k=1

$$(2)C_2(x,t) = ((C_1)_{xx} - \left[\sum_{r=0}^{1}(C_r)\left(C_{(k-r)}\right)_x\right]$$

$$(2)C_2(x,t) = ((C_1)_{xx} - \left[(C_0)\left(C_{(1-0)}\right)_x + (C_1)\left(C_{(1-1)}\right)_x\right]$$

$$= e^{-x} + 4\,e^{-2x} - [(e^{-x})(-e^{-x} - 2\,e^{-2x}) + (\,e^{-x} + \,e^{-2x})(-\,e^{-x})]$$

$$= e^{-x} + 4\,e^{-2x} - [-\,e^{-2x} - 2\,e^{-3x} - \,e^{-2x} - \,e^{-3x}]$$

$$C_2(x,t) = \frac{e^{-x} + 6\,e^{-2x} + 3\,e^{-3x}}{2}$$

Now let k=2,

$$(3)C_3(x,t) = ((C_2)_{xx} - \left[\sum_{r=0}^{2}(C_r)\left(C_{(k-r)}\right)_x\right]$$

$$(3)C_3(x,t) = ((C_2)_{xx} - \left[(C_0)\left(C_{(2-0)}\right)_x + (C_1)\left(C_{(2-1)}\right)_x + (C_2)\left(C_{(2-2)}\right)_x\right]$$

$$= e^{-x} + 24\,e^{-2x} + 27\,e^{-3x}$$
$$- [(e^{-x})(-e^{-x} - 12\,e^{-2x} - 9\,e^{-3x}) + 2.(\,e^{-x} + \,e^{-2x})(-\,e^{-x} - 2\,e^{-2x})$$
$$+ (\,e^{-x} + 6\,e^{-2x} + 3\,e^{-3x})(-\,e^{-x})]$$

$$= \frac{1}{2}[\,e^{-x} + 24\,e^{-2x} + 27\,e^{-3x}$$
$$- [-e^{-2x} - 12\,e^{-3x} - 9\,e^{-4x} + (-2\,e^{-2x} - 4\,e^{-3x} - 2\,e^{-3x} - \,4e^{-4x})$$
$$+ (-e^{-2x} - 6\,e^{-3x} - 3\,e^{-4x})]]$$

$$C_3(x,t) = \frac{1}{6}[\,e^{-x} + 28\,e^{-2x} + 51\,e^{-3x} + 16\,e^{-4x}]$$

In this way we can generated other polynomials by putting different values in equation (7)
Now by inverse Transform

$$u(x,t) = \sum_{k=0}^{r} U_k(x)t^k$$

$$C(x,t) = C_0(x,t)t^0 + C_1(x,t)t \;+ C_2(x,t)t^2 + C_3(x,t)t^3 + \cdots$$

$$C(x,t) = e^{-x}t^0 + (e^{-x} + e^{-2x})t^1 + (\,e^{-x} + 4\,e^{-2x} + 2\,e^{-2x} + 3\,e^{-3x})\frac{t^2}{2!} + (\,e^{-x} + 28\,e^{-2x} + 51\,e^{-3x} + 16\,e^{-4x})\frac{t^3}{3!}\dots$$

VI. RESULTS

The following table shows the approximate values of concentration of impure water at different distance x and time t.

x	t=0	t=0.1	t=0.2	t=0.3	t=0.4	t=0.5	t=0.6	t=0.7	t=0.8	t=0.9	t=1
0	1	1	1	1	1	1	1	1	1	1	1
0.1	0.8494	0.8393	0.8284	0.819	0.8116	0.8059	0.8014	0.7979	0.7951	0.7928	0.7909
0.2	0.7132	0.6971	0.6799	0.6652	0.6536	0.6446	0.6376	0.6321	0.6277	0.6242	0.6212
0.3	0.59	0.571	0.5509	0.5338	0.5231	0.5099	0.5018	0.4955	0.4904	0.4863	0.4828

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0.4	0.4784	0.459	0.4385	0.4212	0.4075	0.397	0.3888	0.3823	0.3772	0.373	0.3696
0.5	0.3775	0.3293	0.3403	0.3241	0.3115	0.3017	0.2941	0.2882	0.2834	0.2796	0.2764
0.6	0.2862	0.2705	0.2541	0.2402	0.2294	0.221	0.2145	0.2094	0.2054	0.2021	0.1994
0.7	0.2036	0.1912	0.1783	0.1674	0.1589	0.1524	0.1473	0.1433	0.1402	0.1376	0.1355
0.8	0.1288	0.1203	0.1114	0.104	0.0982	0.0937	0.0902	0.0875	0.0854	0.0836	0.0822
0.9	0.0612	0.0568	0.0523	0.0486	0.0456	0.0434	0.0416	0.0402	0.0392	0.0383	0.0375
1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

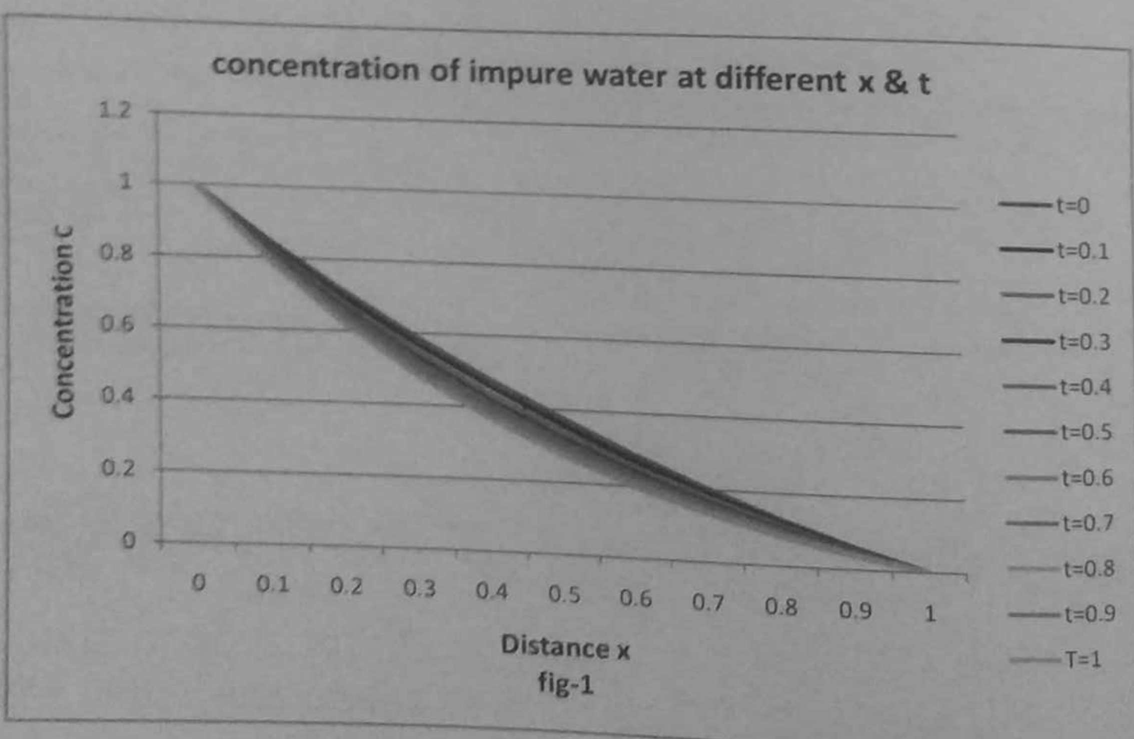

fig-1

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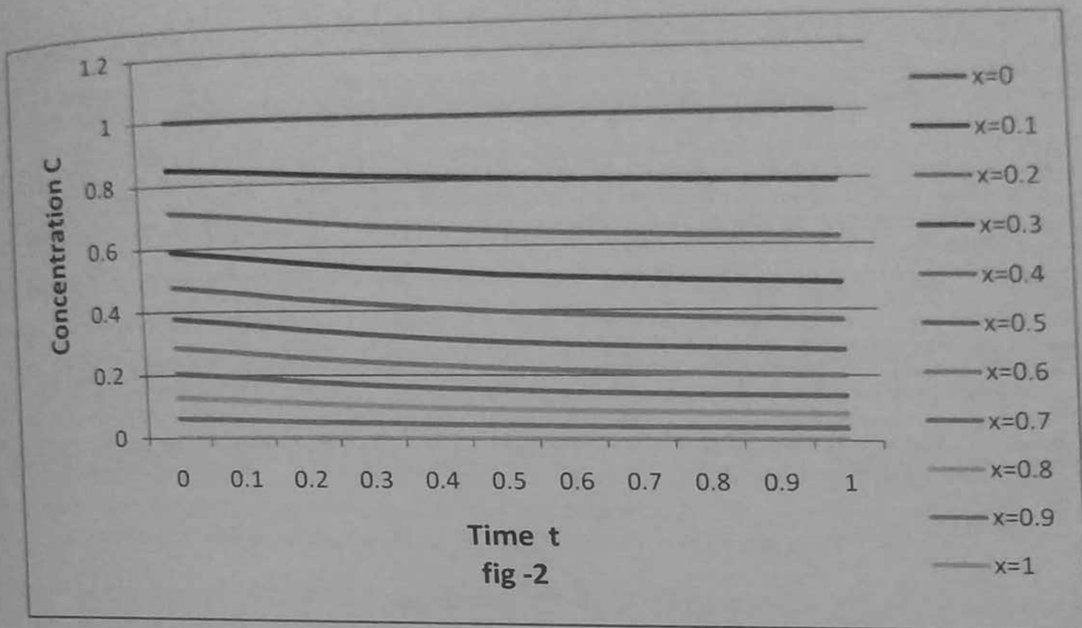

fig -2

VII. CONCLUSION

The graphical and numerical solutions have been obtained to predict the possible concentration of a impure water in unsteady unidirectional seepage flow through semi-finite homogeneous porous media. Focus to the source concentration that vary with the distance x and time $t \geq 0$. From the tabular values and graphs it is conclude that as distance x and time t increases the concentration of the impure water steadily decreases. The concentration C(x, t) of the impure water decreases as the distance x increases for the any fixed time t. Here the initial concentration of impure water at x=0 is highest and it is decreases as distance x increases for fixed time t. It is physically fact that at the source the concentration of impure water is always highest and it is decreasing & dispersing from the source. Here also conclude form the graph (fig-2) of concentration of impure water verses time t for given distance x, the concentration of impure water is decreasing for small time t and then it becomes steady and constant as time t increases for given different distance. Hence, it is clear that at the initial source the dispersion of impure water is not fast, therefore the concentration of impure water is slightly decreasing for small time t, for fixed distance x and then it becomes constant through the time for given distance x.

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